DUSO Mathematics League 2015 - 2016

CONTEST #1.

SOLUTIONS

1 - 1. $\boxed{\mathbf{\frac{14}{3}}}$ Solving, $14 \geq 3x \to x \leq \mathbf{\frac{14}{3}}$.

1 - 2. $\boxed{\mathbf{53}}$ Minimize p by subtracting the greatest unit fraction possible from $\frac{451}{903} < \frac{451}{902} = \frac{1}{2}$, namely $\frac{1}{3}$. The difference $\frac{451}{903} - \frac{1}{3} = \frac{50}{301} < \frac{1}{6}$. Repeat the process, subtracting the greatest possible unit fraction, to obtain $\frac{50}{301} - \frac{1}{7} = \frac{1}{43}$. Therefore, the desired sum is $3 + 7 + 43 = \mathbf{53}$.

---

1 - 3. $\boxed{\mathbf{124}}$ Compute $m\angle TRA = m\angle TRG + m\angle GRA$. First, $m\angle TRG = 112 \div 2 = 56$ because $\angle RGM$ is an exterior angle to $\triangle TRG$ and $\angle RGM \cong \angle RAM$. Next, $m\angle GRA = 180 - 112 = 68$. Thus, $m\angle TRA = 56 + 68 = \mathbf{124}$.

1 - 4. $\boxed{\mathbf{8 + 4\sqrt{2}}}$ Extend $\overline{AM}$ and drop a perpendicular from S to $\overleftrightarrow{AM}$ with foot U. Note that $m\angle AMS = \frac{6 \cdot 180^\circ}{8} = 135^\circ$, so $m\angle SMU = 45^\circ$, and thus $MU = SU = \sqrt{2}$. Now apply the Pythagorean Theorem to $\triangle SMU$ to obtain $\sqrt{2}^2 + (2 + \sqrt{2})^2 = AS^2 = [ARCS]$. Thus, $[ARCS] = 2 + (4 + 2 + 4\sqrt{2}) = \mathbf{8 + 4\sqrt{2}}$.

---

1 - 5. $\boxed{\mathbf{22}}$ Factor to obtain $x^2(x-2) - 9(x-2) = (x^2-9)(x-2) = (x-3)(x+3)(x-2)$. The three solutions are -3, 3, and 2, so the desired quantity is $9 + 9 + 4 = \mathbf{22}$.

1 - 6. $\boxed{\mathbf{\frac{\sqrt{97}}{4}}}$ The desired difference is $\frac{p-q}{pq}$. The sum of the roots of the quadratic $x^2 - 9x - 4 = 0$ is 9, and the product of the roots is -4. Note that $(p+q)^2 = 81 = p^2 + q^2 + 2pq \to p^2 + q^2 = 89$. The desired quantity can be found by recognizing that $(p-q)^2 = p^2 + q^2 - 2pq = 89 - 2(-4) = 97$, so $p - q = -\sqrt{97}$ (since $p < q$). The value of $\frac{p-q}{pq}$ is $\frac{-\sqrt{97}}{-4} = \mathbf{\frac{\sqrt{97}}{4}}$.

Author: George Reuter - coachreu@gmail.com – Reviewer: Michael Curry - currymath@gmail.com

R-1. If Sally subtracts 2 from her locker number, and then multiplies the result by 3, and then adds 5, the result is 2015. What is Sally's locker number?
R-1Sol. $\boxed{\mathbf{672}}$ Solve $3(x-2)+5=2015$ to obtain $x = \mathbf{672}$.

R-2. *Let N be the number you will receive.* The line $2x+3y=N$ has an x-intercept of A and a y-intercept of B. Compute $A+B$.
R-2Sol. $\boxed{\mathbf{560}}$ The x-intercept is $\dfrac{N}{2}$ and the y-intercept is $\dfrac{N}{3}$, so $A+B=\dfrac{5N}{6}$. Substituting, $A+B=5\dfrac{672}{6}=5(112)=\mathbf{560}$.

R-3. *Let N be the number you will receive.* In a rectangle, the sum of two lengths and a width is 820. The sum of two widths and a length is N. Compute the perimeter of the rectangle.
R-3Sol. $\boxed{\mathbf{920}}$ This problem is equivalent to $2L+W=820$ and $2W+L=N$. Adding the equations, $3W+3L=820+N$. Because the question asks for the perimeter, or $2L+2W$, multiply by $2/3$ to obtain $P=\dfrac{2(820+N)}{3}$. Substituting, $P=\mathbf{920}$.

R-4. *Let N be the number you will receive.* In a video game, getting 1 red turtle, 2 green turtles, and 3 blue turtles earns 1050 points. In the same game, getting 3 red turtles, 4 green turtles, and 1 blue turtle earns 910 points. In the same game, earning 5 red turtles, 3 green turtles, and 5 blue turtles earns N points. If Jimmy gets 1 red turtle, 1 green turtle, and 1 blue turtle, how many points does he earn?
3-4Sol. $\boxed{\mathbf{320}}$ Adding the three equations $1R+2G+3B=1050$, $3R+4G+1B=910$, and $5R+3G+5B=N$ obtains the result $9R+9G+9B=1960+N$, so $R+G+B=\dfrac{1960+N}{9}$. Substituting, $R+G+B=\mathbf{320}$.

R-5. *Let N be the number you will receive.* The line $-16x+40y=N$ passes through many points in the second quadrant, but only some of those have integer coordinates. Compute the number of points in the second quadrant that have integer coordinates and are also on the line $-16x+40y=N$.
R-5Sol. $\boxed{\mathbf{3}}$ The line has a slope of $\frac{2}{5}$ and has an x-intercept of $\frac{N}{-16}=-20$. Therefore, moving right 5 and up 2 from that x-intercept will give lattice points. This can be done until $x=0$, or **3** times.